Quantum Chess, Reprinted with permission from The Pentagon, VXXVII, No.2, Spring 1968, pp 99-103, Kappa Mu Epsilon student math magazine. (Reduces moves to simple equations on a six by eight chessboard with 5 basic piece shapes and eight total pieces per player.)

Quantum Chess

DOUGLAS A. ENGEL
Climax Molybdenum Co., Climax, Colorado

Today a great many chess games are played between computers and men. No computer program has yet been devised that wins every time against mortal man, but there are some that come close. Perhaps man is aided by the fact that he has been playing for over a thousand years. At the same time it is man who has created that diabolic contraption, the computer. So if he begins to lose every time he can only blame himself.

Chess is a highly mathematical game. In fact the moves of several of the pieces can be described by linear equations in x and y. For instance the rook can move any distance orthogonally in two directions. If one thinks of the rook's square as an origin equal to zero, then the moves of the rook are described by the equations $x = n, y = 0$, and $x = 0, y = n$. After the rook is moved the square upon which it resides is a new origin equal to zero and the same set of equations applies to its future movement. One can describe the moves of the bishop by the equation $y = x$, where both directions of x and y are positive. Again the square upon which the bishop stands must be the zero point at all times for this equation to hold. The queen, of course, moves according to both the equations for the rook and the equation for the bishop. The knight moves according to the equation $y = x^2 + 1$, where the only value x can take is 1. Hence the knight can move from the square it stands upon any combination of the two orthogonal directions 1 and 2. A simpler equation could have been given for the knight, but the above one was given in order to show how one could see some of the pieces as being described by a possibly more complex system than is at first apparent.

The highly mathematical moves of the chessmen suggest that one could vary the game to take advantage of an even more purely mathematical structure. One could add constants to the variables and otherwise change the game so that the moves are recognizd more for their equations than the word rule for them. This change could result in a game in which one would write a set of equations and then play the game.

A simple version of chess changed by changing the equations of motion of the pieces is played on a 6 by 8 board. The equations of motion of the pieces and some of the possible moves are shown in Fig. 2. Figure 1 shows a suggested opening setup for beginning the game. The author has named this game quantum chess because only certain integral values are allowed x and y.

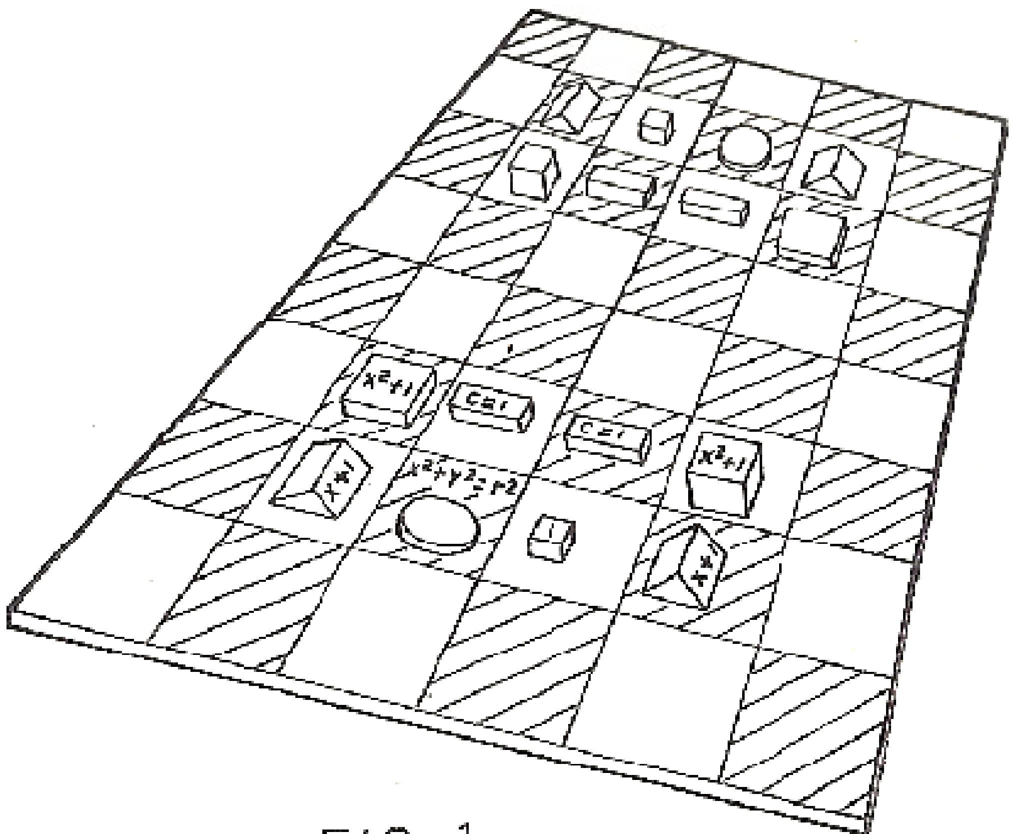

FIG. 1

In Fig. 2 the piece $x^2 + 1$ can move any combination of the distances (0,1), (1,2), (2,5), and (3, square on border). A free path must be open for any of these moves to take place. If a man is captured he can only be captured at the endpoint of the move. For instance, the $x^2 + 1$ piece in Fig. 2 can move to the right 3 squares and down to the border where a dot is placed to indicate this move. The move can take place if either the path down and to the right or to the right and down is open. The direction taken for x or y is, of course, arbitrary, but the x and y directions are always perpendicular. Since the x and y (positive) axes are arbitrary each equation describes moves that are similar in each of the four quadrants. One of the curves described by the piece $x^2 + 1$ is shown by the curved line in Fig. 2.

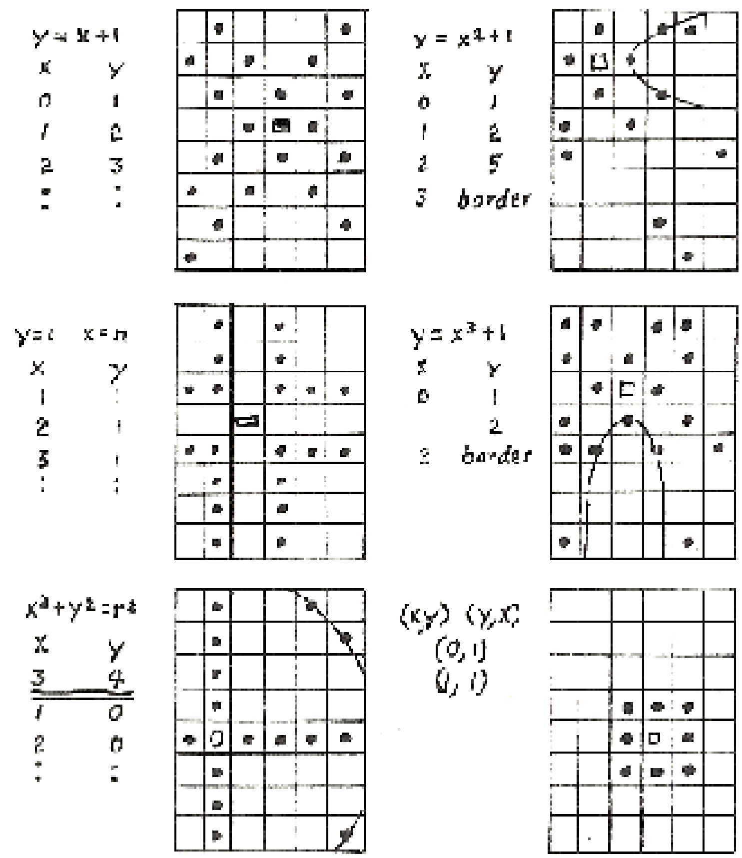

FIG. 2

One of the pieces shown in Fig. 2 obeys the equation for a circle. Since this equation has integral solutions if either x or y is zero the piece can move any orthogonal distance in either direction like a rook. In addition the circle can move any combination of the two perpendicular distances 3 and 4, because 3 and 4 satisfy the equation,

$$3^2 + 4^2 = 5^2$$

The unit piece moves just like the king in chess. The game is won by capturing the unit. There are no pieces equivalent to the pawn as in chess.

The terminology 'annihilated' has been added for a piece that is captured. Strictly speaking annihilation would have to destroy both pieces if any analogy with atomic particles were to hold. Player and antiplayer can begin with the opening setup shown in Fig. 1. Some shapes for pieces are also suggested in Fig. 1. If you have a chessboard you can place a strip of paper over the 2 by 8 portion of squares that is not used for quantum chess. Since there are six basically different pieces in chess you can use these for pieces of quantum chess if you want to. However, it is easy to cut the pieces shown in Fig. 1 out of a square bar of wood. It will also be much less confusing since pieces logically resemble their equations of motion and will be easily remembered if they have the shapes shown.

On the opening moves the $C = 1$ piece ($x = n$, $y = 1$) cannot move up and down but can move sideways. This rule is added so that a man cannot be captured on the first move. It appears that in the first few moves the man to move first usually has a small advantage. As the game progresses it appears that the advantage disappears, as in chess. The pieces seem to be quite powerful but it can be very difficult to checkmate, even if you have a good advantage. The game develops faster than chess, but rapidly develops complexity so that a player may find himself taking quite a long time to decide on a move. There is also an advantage in quantum chess in learning the moves of the various pieces as equations. This gives the game a distinct educational value and allows an objective approach. One will soon attempt tricky moves from the logical standpoint of solutions to several equations after transforming. It is possible to develop a good deal of respect and facility for cartesian coordinates by merely playing this game, or other versions of it.

Adding A Dimension

The quadruple symmetry of the moves of the various pieces in quantum chess makes it easy to add a dimension of time to the moves. Up and down directions may be considered to belong to the $y-$ axis while sideways directions are the $x-$ axis. On a player's first move all pieces obey the equations $y = x - 1$, ($y = 1$), $x = 1, 2, \cdots$); $y = x^2 + 1$, $y = x^3 + 1$, and the given equation for the circle. On the same player's second move all pieces obey the same equations with x and y interchanged. On all odd moves, first, third, fifth, that a player makes his pieces can move only according to the equations given above. On all even moves pieces must obey the same equations with x and y interchanged, as stated above for a second move.

Thus time enters into the game using the idea of odd and even parity moves. The time rule alternately rotates the strategy of the game 90° so that one must keep thinking in terms of a change of parity on the next allowable moves. At the same time the power of the pieces diminishes so that the total complexity of the game stays about the same. However, since power has diminished in the pieces it will take longer to play a game with the time rule added. Captures will tend to occur with less frequency through the same number of moves.

Hopefully students of mathematics will take up quantum chess and develop their own versions so that they may enjoy many games while learning valuable mathematics.

References

1. Maurice Kraitchik, *Mathematical Recreations*, (New York: Dover, 1953), pp. 267-323.
2. Joseph S. Madachy, *Mathematics on Vacation*, (New York: Charles Scribner's Sons, 1966), pp. 34-54.
3. Martin Gardner, *Martin Gardner's New Mathematical Diversions from Scientific American*, (New York: Simon and Schuster, 1966), pp. 70-81.