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257. No Calculus, Please! by Douglas Engel, Denver, Colorado (*J. Recreational Math.*, 6(1), Winter 1973, p. 72)

Find the formula for the volume of a torus with the inner half cut out, given the formula for the volume of a sphere: $4\pi r^3/3$.

Solution: We use a *Theorem of Pappus:* The volume of a solid of revolution, formed by revolving a plane area about a line in its plane not cutting the area, is equal to the product of the generating area and the circumference of the circle described by the centroid of the area.

The shaded area in Figure 7 represents the cross-section view of the half torus. Figure 8 shows the same view, without the intervening toroidal hole. The volume of the solid in Figure 8 is the same as the volume of a sphere with radius r: $4\pi r^3/3$.

Figure 7.

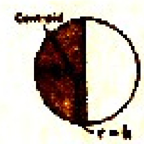

Figure 8.

By the Theorem of Pappus this is also the volume generated by the product of the area of a semicircle with radius r and the circumference of the circle with radius $r - k$ (the distance of the centroid from the center of the circle).

The volume of a sphere is given, so the centroid location, $r - k$, can be calculated:

$$\text{Volume of sphere} = (\text{Area of semicircle})(r - k)(2\pi)$$
$$4\pi r^3/3 = (\pi r^2/2)(r - k)(2\pi)$$
$$r - k = \frac{4\pi r^3/3}{(\pi r^2/2)2\pi} = \frac{4r}{3\pi}$$

Then $R + (r - k)$ is the distance of the centroid of the generating area to the center of the half torus in Figure 7. By Theorem of Pappus, then,

$$\text{Volume of half torus} = (\text{Area of generating area})(R + r - k)$$
$$= (\text{Area of semicircle})(R + 4r/3\pi)$$
$$= (\pi r^2/2)(R + 4r/3\pi)$$
$$= \pi^2 R r^2 + \frac{4\pi r^3}{3}$$

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178. *Proposed by Douglas A. Engel, Hays, Kansas.*

Prove that the following formula is true:

$$n! = (n-1)(n-1)! + (n-2)(n-2)! + \cdots + 2(2!) + 1(1!) + 1(0!).$$

Solution by Paul M. Flynn, Kansas State College of Pittsburg, Pittsburg, Kansas.

The proof is by finite induction. For $n = 1$

$$1(0!) = 1 = 1!$$

Assume the proposition true for $n = k$.

$$1(0!) + (1)(1!) + 2(2!) + \cdots + (k-2)(k-2)! + (k-1)(k-1)! = k!$$

Then we have for $n = k + 1$

$$1(0!) + (1)(1!) + \cdots + (k-1)(k-1)! + [(k+1)-1][(k+1)-1]!$$

$$= k! + [(k+1)-1][(k+1)-1]!$$

$$= k! + k(k!) = (k+1)k! = (k+1)!$$

Since the proposition is true for $n = 1$ and is true for $n = k + 1$ when it is true for $n = k$, the proposition is true for all positive integers.

Also solved by Harold Darby, Florence State College, Florence, Alabama, John L. Lebbert, Washburn University, Topeka, Kansas, LeRoy Simmons, Washburn University, Topeka, Kansas, and the proposer.

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Several replies did not prove this but advanced the sequences. Not known if it was ever proved since then.

886.* [November, 1973] *Proposed by Doug Engel, Denver, Colorado.*

In a sequence of positive integers, $N_{k+1} = N_k +$ the sum of all the distinct prime factors of N_k including 1 and N_k, $k = 0, 1, 2, \cdots$. Such a sequence is 1, 2, 3, 4, 7, 8, 11, 12, 18, 24 ⋯. The sequence 5, 6, 12, 18 ⋯ merges with the first sequence as do all sequences with $N_0 < 91$.

It is conjectured that such sequences with N_0 a positive integer merge with the basic sequence that begins with 1. Prove or disprove this conjecture. If disproven, how many independent sequences exist?

Editor: No solutions received to this problem. C. W. Trigg comments that all $N_0 < 105$ merge before or at the 24th term of the basic sequence except the sequence for 91 merges at the 38th term. We would welcome further comments.