Reprinted with Permission from the **Journal of Recreational Mathematics** and **Baywood Periodicals,** V 2, Nno. 2 April, 1969. (Blurb about how some idea in mathematics become major fields and powerful tools such as negative numbers, imaginary numbers....)

How to Get Something for Nothing

Douglas A. Engel
Denver, Colorado

By adding certain quantities together and then by certain other operations it is possible to prove that one can get something for nothing. For instance, if we add negative numbers to the positive numbers we are all familiar with (we all have a positive amount of money, of course) we get a completely new and surprising thing called i or the square root of minus 1. To prove that this is so merely ask yourself how it is possible to multiply two identical numbers together and get -1? Obviously the only way to do this is with i. This, of course, results in the very interesting addition of imaginary numbers. Imaginary numbers are numbers of the form $a + bi$. But for a long time mathematicians were confused by imaginary numbers and the paradoxical i. Thus, although they got something for nothing (indeed a whole mathematical system) they claimed it was forced on them and a long battle of fighting the i ensued which of course resulted in a great deal of complication of certain branches of mathematics.

The reader knows the commandment "Thou shalt not $n/0$." However, what about the case $0/0$? Surely it must be equal to 1. But if $(0)(n) = 0$, then $0/0 = n$. Thus we have just gained a great deal for nothing by disobeying the given commandment. This is a serious sin because it determines nothing and shows that 0, being nothing, is probably the freest actor in all mathematics. We might wish 0 to be the least free and most tied down one in the number kingdom. This difficulty was never resolved until the science of infinite limits was worked out. Since 0 is infinitely small it at least can be related to the word infinite. The science of mathematics has never really recovered from this complex blow still being dealt by the transfinites.

The Pythogoreans were quite familiar with the terrible truth of getting something for nothing when you least want it. For instance, it can be shown that $\sqrt{2}$ cannot be written down with ordinary integers and must be written $\sqrt{2}$. This was a terrible blow to simplicity but it eventually added the whole branch of numbers

Some other low blows came with the discovery of transcendental numbers (a new set of points on the number line). These seemed to gravitate to areas that were thought most perfect such as the circle and sphere, and such natural things as growth. A transcendental number cannot be written down as a finite number of irrational expressions. Hence we have such things as π and e.

Further, it can be proven that you certainly can have your pi and e at it too. For by throwing away an axiom about parallels we are told (in a very austere and mysterious tone) that there is a multitude of geometries. Now we know that the shortest distance between two points is not necessarily straight. It is really the path of least resistance. Thus while someone is eating Euclidean geometry he can be traversing around it and having his Riemannian or Lobachevskian geometry. Though it is questionable whether anyone in his right mind would want to do this.

Thus it has been proven for us that it is indeed possible to get something for nothing. At least it is proven that one usually gets more than he asked for. It's the nearest thing to getting something for nothing that we may ever be able to approximate to. We may profit by the experience of our ancestors and whenever we come upon something irreconcilable with anything presently known in mathematics accept that we may have opened a new door and furthermore be careful not to condemn its willingness to give, for it isn't often that we get something for nothing.