Flexing Rings of Regular Tetrahedra, **R**eprinted with permission from ***The Pentagon***, a Kappa Mu Epsilon student mathematical publication, 1967, pp 106-108, 122. *(Some of my research into topological properties of digital twist.)*

Flexing Rings of Regular Tetrahedra

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A tetrahedral ring is formed by connecting opposite edges of several tetrahedra to form a circular, hinged, chain [1]. The purpose of this paper is to show how rings of regular tetrahedra can be formed with cyclic properties of movement. If an element (regular tetrahedron) of a ring is held non-rotatable in space and the ring can be rotated about it any number of times, the ring is *flexible*. A ring of eleven elements, with a ¾ twist between its ends, has an interesting cycle of flexations and a unique symmetry.

Two chains of regular tetrahedra can be wound about one another to form a solid core (Fig. 1). A chain that is given a simple

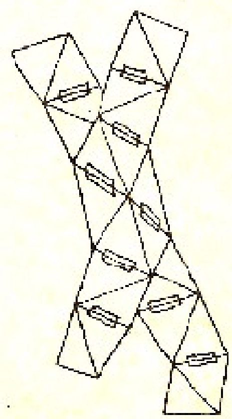

Fig. 1

twist forms a core with a shape congruent to that of two chains wound about one another, but it is not rigid. Four of the rigid cores can be connected at an intersection to give a structure that flexes by winding the opposite cores into the adjacent cores (Fig. 2). This

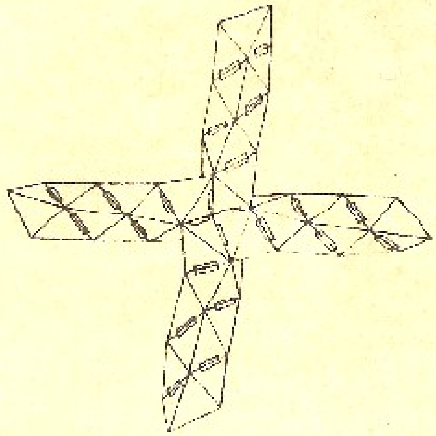

Fig. 2

structure alone is not a true flexible solid, since any number of rotations of the ring about an element are not possible. By leaving a loose portion in each of the four arms they can be reconnected into a four armed structure (Fig. 3). The resulting structure is flexible.

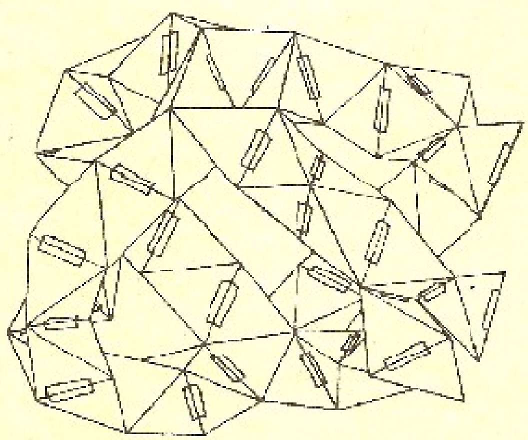

Fig. 3

A flexing structure type can be formed by winding two chains about one another for a short ways and then winding one of the chains out, around the other, before continuing the winding process (Fig. 4). The flexing motion is a simple rolling of the

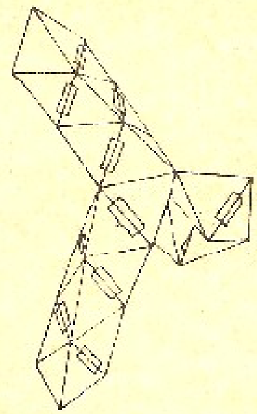

Fig. 4

tetrahedra in the protruding portion, in either direction, along the ring. The protruding structure bends the cores about 30°, allowing a circular chain of a number of protruding structures which is a true flexing solid. The loose portions in the structure in Fig. 3 can be replaced by a number of the structures in Fig. 4, giving a flexible solid that is rigid except for its flexing movement. More complex solids can be formed by having more four armed structures in the ring. The number of twists between the ends of two chains wound about one another is approximately $\frac{n}{4}$, where n is the total number of elements. Regular tetrahedra in a linear core can be flattened to a layer of squares. The four armed structure can be reduced to a general type of tetraflexagon by flattening all four cores [2] [3].

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